Gluon spectral densities from the lattice

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Outline

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Motivation

- Main goal: compute the spectral density of gluons and other (un)physical degrees of freedom
  - important for e.g. DSE/BSE spectrum studies (Minkowski space)
  - spectral density is not strictly positive
  - traditional Maximum Entropy Method does not allow negative spectral densities

D. Dudal, O. Oliveira, PJS, PRD 89 (2014) 014010

Positivity violation

Spectral representation

$$D(p^2) = \int_0^{+\infty} d\mu \frac{\rho(\mu)}{p^2 + \mu^2}$$

On the lattice: study the temporal correlator

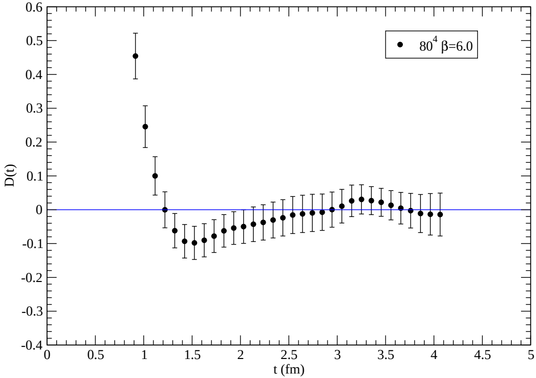
$$C(t) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} D(p^2) \exp(-ipt) = \int_0^{\infty} d\omega \rho(\omega^2) e^{-\omega t}$$

$C(t) < 0$

- negative spectral density
- positivity violation
- gluon confinement

$C(t) > 0$ says nothing about $\rho(\mu)$

Positivity violation for the gluon propagator



Already observed in lattice simulations

C. Aubin, M. C. Ogilvie, Phys. Rev D70, 074514 (2004)

A. Cucchieri, T. Mendes, A. R. Taurines, Phys. Rev. D71, 051902 (2005)

Spectral density

- Euclidean momentum-space propagator of a (scalar) physical degree of freedom

$$\mathcal{G}(p^2) \equiv \langle \mathcal{O}(p)\mathcal{O}(-p) \rangle$$

- Källén-Lehmann spectral representation

$$\mathcal{G}(p^2) = \int_0^\infty \mathrm{d}\mu \frac{\rho(\mu)}{p^2+\mu}\,, \qquad \text{with } \rho(\mu) \geq 0 \text{ for } \mu \geq 0\,.$$

- spectral density contains information on the masses of physical states described by the operator $\mathcal{O}$

$$\rho(\mu) = \sum_\ell \delta(\mu - m_\ell^2)\, |\langle 0|\mathcal{O}|\ell_0\rangle|^2\,,$$

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Spectral density

- $\mathcal{G} = \mathcal{L}^2\hat{\rho} = \mathcal{L}\mathcal{L}^*\hat{\rho}$ where $(\mathcal{L}f)(t) \equiv \int_0^\infty ds e^{-st} f(s)$ is a Laplace transform
- inversion of Laplace transform: ill-posed problem
- Way out: Tikhonov regularization
  - ill-posed problem $y = \mathcal{K}x$
  - minimize $||\mathcal{K}x - y|| + \lambda||x||^2$
    - $\lambda > 0$ is a regularization parameter
  - x^λ is the unique solution of the normal equation

$$\mathcal{K}^*\mathcal{K}x^\lambda + \lambda x^\lambda = \mathcal{K}^* y$$

  the operator $\mathcal{K}^*\mathcal{K} + \lambda$ is strictly positive, hence invertible
- Morozov discrepancy principle: choose $\bar{\lambda}$ s.t. $||\mathcal{K}x^{\bar{\lambda}} - y^\delta|| = \delta$
  - δ: “noise of input data”
  - A unique solution $x^{\bar{\lambda},\delta}$ exists

Outline

How-to

$$\mathcal{G}(p^2) = \int_{\mu_0}^{+\infty} \mathrm{d}\mu \frac{\rho(\mu)}{p^2+\mu},$$

- Infrared threshold μ_0 (to be determined)
- $\mathcal{G}_i \equiv \mathcal{G}(p_i^2)$, N data points, we will minimize

$$\mathcal{J}_\lambda = \sum_{i=1}^{N} \left[\int_{\mu_0}^{+\infty} \mathrm{d}\mu \frac{\rho(\mu)}{p_i^2+\mu} - \mathcal{G}_i \right]^2 + \lambda \int_{\mu_0}^{+\infty} \mathrm{d}\mu\, \rho^2(\mu)$$

- perturbing $\rho(\mu)$ linearly
- demanding the vanishing of variation of $\mathcal{J}_\lambda$

$$\sum_{i=1}^{N} \underbrace{\left[\int_{\mu_0}^{+\infty} \mathrm{d}\nu \frac{\rho(\nu)}{p_i^2+\nu} - \mathcal{G}_i \right]}_{\equiv c_i} \frac{1}{p_i^2+\mu} + \lambda\rho(\mu) = 0 \ (\mu \geq \mu_0)$$

How-to 2

- (regularized) solution to KL inversion:

$$\rho_\lambda(\mu) = -\frac{1}{\lambda}\sum_{i=1}^{N}\frac{c_i}{p_i^2+\mu}\theta(\mu-\mu_0)\,,$$

- threshold crucial to avoid a singularity at $\mu = 0$

$$c_i = -\frac{1}{\lambda}\int_{\mu_0}^{+\infty} \mathrm{d}\nu \frac{1}{p_i^2+\nu}\sum_{j=1}^{N}\frac{1}{p_j^2+\nu}c_j - \mathcal{G}_i\,,$$

How-to 3

$$\lambda^{-1}\mathcal{M}c + c = -\mathcal{G}\,,$$

$$\mathcal{M}_{ij} = \int_{\mu_0}^{+\infty} d\nu \frac{1}{p_i^2+\nu}\frac{1}{p_j^2+\nu} = \frac{\ln\frac{p_j^2+\mu_0}{p_i^2+\mu_0}}{p_j^2-p_i^2}\,.$$

- $\mathcal{M}_{ii} = 1/(p_i^2+\mu_0)$
- perfectly well-defined, symmetric matrix for $\mu_0 > 0$
- inverse KL operation reduced to solving a linear system
- reconstructed propagator:

$$\mathcal{G}_\lambda(p^2) = \int_{\mu_0}^{+\infty} d\mu \frac{\rho_\lambda(\mu)}{p^2+\mu} = -\frac{1}{\lambda}\sum_{i=1}^{N}\frac{c_i \ln\frac{p^2+\mu_0}{p_i^2+\mu_0}}{p^2-p_i^2}\,.$$

Test

Consider a (non-relativistic) “Breit-Wigner” toy spectral density with nonzero threshold,

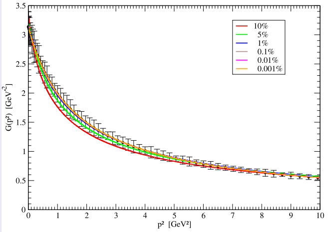
$$\rho(\mu) = \frac{\mu}{(\mu - m^2)^2 + \Gamma^2/4}\theta(\mu - \overline{\mu}) \tag{1}$$

$$m^2 = 1\ \text{GeV}^2\,, \Gamma = 1\ \text{GeV}^2\,, \overline{\mu} = 0.1\ \text{GeV}^2\,.$$

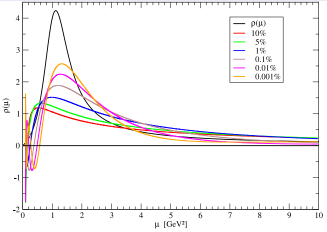
- propagator computed using a Gauss-Legendre quadrature with 1000 points and $\sqrt{\mu}_{max} = 20$ GeV
- to get the reconstructed spectral density: Gauss-Jordan normal elimination, $N = 120$ entry data points.
  - We assigned to each data point $\mathcal{G}_i$ the percent errors $\varepsilon = 10, 5, 1, 0.1, 0.001, 0.0001$ according to $\mathcal{G}_i \times \varepsilon \times (0.5 + 0.5r)$, with r a uniform random number $\in [0, 1]$.

Test

Input propagators and reconstructions



Spectral functions



Outline

1. Introduction and Motivation
2. New method to compute spectral densities
   - How-to
   - Landau gauge gluon propagator
   - Spectral density at finite T
3. Conclusions and outlook

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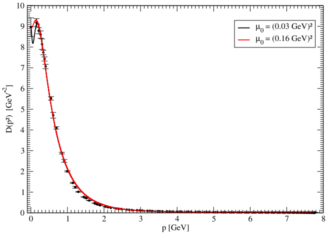
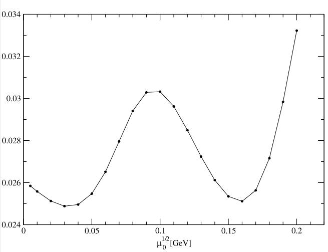
Real case

Landau gauge SU(3) Yang-Mills gluon propagator 80^4, $\beta = 6.0$

Input propagator and reconstructions

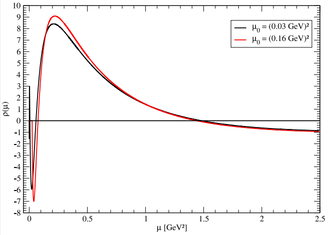


λ as a function of μ_0

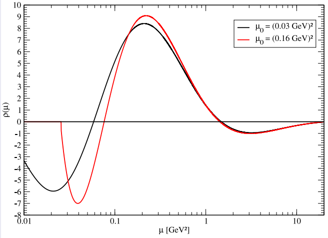


Gluon spectral function

Linear scale



Log scale



Outline

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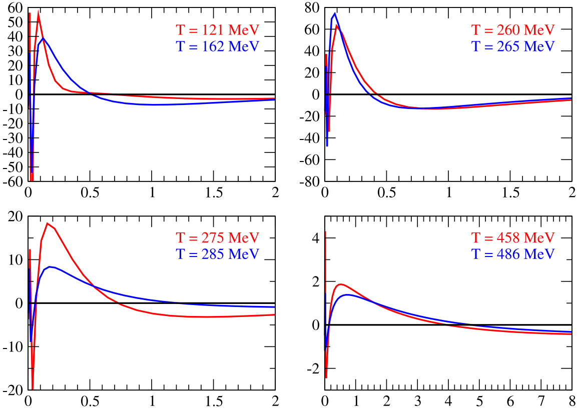
Longitudinal propagator spectral densities

Preliminary results!



Conclusions and outlook

- Method to compute spectral densities
  - does not rely on the *a priori* positivity of $\rho(\mu)$
- Results for the Landau gauge gluon propagator
- Preliminary results for finite temperature
  - Positivity violation scale increases with temperature
  - Gluons behave as quasi-particles for high T?
- Outlook
  - finite temperature ($T \sim T_c$)
  - Landau gauge ghost propagator
  - physical SU(3) lattice scalar glueball

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